

Barem de corectare CMAA 2026 Clasa a XII-a – Științe ale naturii

P1 (S.G.M. nr. 10 -2025)

a) $\int \frac{x}{1+x+e^x} dx = \int \frac{1+x+e^x-1-e^x}{1+x+e^x} dx = \int \left(1 - \frac{1+e^x}{1+x+e^x} \right) dx =$	5p
$= x - \int \frac{(1+x+e^x)'}{1+x+e^x} dx = x - \ln(1+x+e^x) + C, C \in \mathbb{R}$	5p
b) Scriem $\int \frac{1}{x^3+x^7} dx = \int \frac{1+x^4}{x^3+x^7} dx - \int \frac{x^4}{x^3+x^7} dx =$	5p
$= \int \frac{1+x^4}{x^3(1+x^4)} dx - \int \frac{x^4}{x^3(1+x^4)} dx = \int \frac{1}{x^3} dx - \int \frac{x}{1+(x^2)^2} dx =$	5p
$= -\frac{1}{2x^2} - \frac{1}{2} \arctg x^2 + C, C \in \mathbb{R}$	5p

P2

a) $f(x) * f(y) = \frac{\frac{x}{x+1} \cdot \frac{y}{y+1}}{2 \frac{x}{x+1} \cdot \frac{y}{y+1} - \frac{x}{x+1} - \frac{y}{y+1} + 1} = \frac{xy}{2xy - x(y+1) - y(x+1) + (x+1)(y+1)} =$	5p
$= \frac{xy}{xy+1} = f(xy), \forall x, y \in (0, \infty)$	5p
b) Fie $t = f(x) \in (0, 1)$. Avem $f(x) * f(x) * f(x) = \frac{8}{9} \stackrel{a)}{\Leftrightarrow} f(x^3) = \frac{8}{9}$	5p
$\frac{x^3}{x^3+1} = \frac{8}{9} \Leftrightarrow x = 2$	5p
$t = f(2) = \frac{2}{3} \in (0, 1)$	5p

P3

a) $P(x) = 30x - x^2 + C$	5p
$P(10) = 250 \Leftrightarrow 300 - 100 + C = 250 \Rightarrow C = 50 \Rightarrow P(x) = 30x - x^2 + 50$	5p
b) $P'(x) = 0 \Rightarrow x = 15$	5p
Valoarea maximă este $P(15) = 275$	5p

P4

$I = \int_0^4 f(x) dx = \frac{1}{2\sqrt{2}} \int_0^1 \sqrt{x^2+1} dx + \int_1^4 \frac{x}{1+\sqrt{x}} dx$	3p
$I_1 = \int_0^1 x' \sqrt{x^2+1} dx = \left(x \sqrt{x^2+1} \right) \Big _0^1 - \int_0^1 \frac{x^2+1-1}{\sqrt{x^2+1}} dx = \sqrt{2} - I_1 + \left[\ln \left(x + \sqrt{x^2+1} \right) \right] \Big _0^1$	4p
$I_1 = \frac{1}{2} \left(\sqrt{2} + \ln(1+\sqrt{2}) \right)$	3p
$I_2 = \int_1^4 \frac{x}{1+\sqrt{x}} dx \stackrel{x=t^2}{=} \int_1^2 \frac{t^2}{1+t} \cdot 2t dt = 2 \int_1^2 \frac{t^3+1-1}{1+t} dt$	4p
$I_2 = 2 \int_1^2 \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt = \frac{11}{3} + \ln \frac{4}{9}$	3p
$I = \frac{1}{2\sqrt{2}} I_1 + I_2 = \frac{1}{4\sqrt{2}} \left(\sqrt{2} + \ln(1+\sqrt{2}) \right) + \frac{11}{3} + \ln \frac{4}{9} = \frac{47}{12} + \frac{1}{4\sqrt{2}} \ln(1+\sqrt{2}) + \ln \frac{4}{9}$	3p

